

## Reliability Design of Mechanical Systems Subject to Repetitive Stresses

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### Abstract

The basic reliability concepts-parametric ALT plan, failure mechanism and design, accelerated testing with action plans, and checking if product achieves the reliability targets-were used in the development of a parametric accelerated life testing method to improve the reliability of mechanical systems subjected to repetitive stresses. To calculate the acceleration factor of the mechanical system, a generalized life-stress failure model with a new effort concept was derived. The new sample size equation with the acceleration factor also enables engineer to uncover the faulty designs affecting reliability during the design process of the mechanical system. Consequently, it might help companies to improve product reliability and avoid recalls due to the product failures in the field. As the improper product designs are experimentally identified by this new reliability design method, the mechanical system might improve its reliability. As case study, we have studied the reliability of a newly designed Freezer drawer. In the first ALT, the handles were being fractured because of design flaws due to the repetitive opening/closing with the food loads. As the total handle width increases, the handle design was corrected. In the second ALT, the slide rails also fractured because they did not have enough strength to withstand repetitive opening and closing of the drawer with the food loads. Additional reinforced ribs, reinforced boss, and an inner chamber in slide rails were provided to improve the design of the slide rails.

**Keywords:** Failure mechanics and design, Life-stress model, Acceleration factor, Sample size equation, Parametric ALT.

### Introduction

Reliability describes the ability of a system or module to function under stated conditions for a specified period of time. Generally, the reliability described by the failure rate over the product life, can be characterized in time by three phases. First, there is a decreasing failure rate, then a relatively constant failure rate, and then an increasing failure rate. These phases are illustrated in diagram called "the bathtub curve" shown in the top curve in Figure 1.

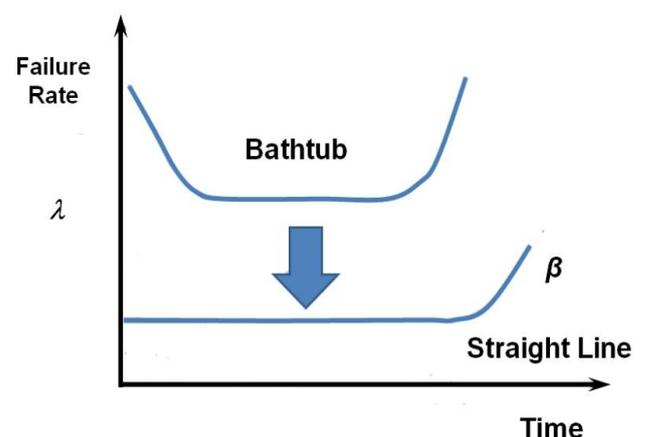


Figure 1: Bathtub curve and straight line with slope  $\beta$  toward the end of the life of the product.

A product that follows the bathtub curve will have difficulties in succeeding in the market. Because of the higher failure rates in the early years of operation, the product will potentially incur financial losses for the company and reduce consumer confidence in the product due to perceived poor quality. If excessive reliability problems persist, the company may be required to remove the product from the market. The company will then need to set goals for new products to (1) reduce early failures, (2) decrease random failures during the product operating time, and (3) increase product lifetime.

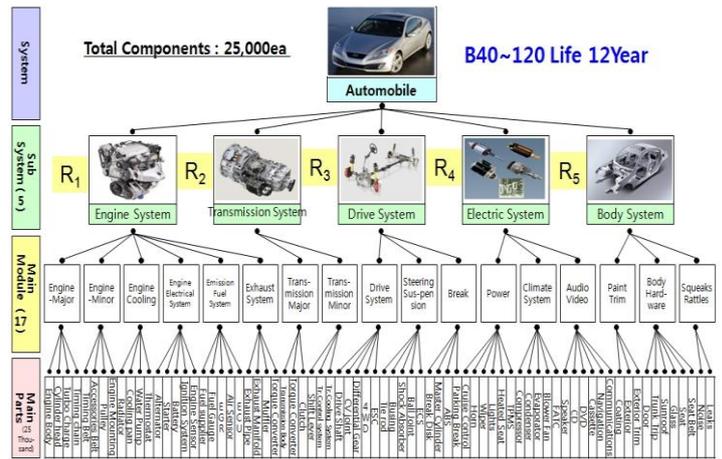
As the reliability of a product is improved through systematic testing, the failure rate of the product in the field would decline. For such a situation, the traditional failure rate typified by the bathtub curve can be reduced to resemble the failure rate represented by a flat, straight line with the shape parameter  $\beta$  in Figure 1. The product reliability might be quantified from the product lifetime  $L_B$  and failure rate  $\lambda$  as follows:

$$R(L_B) = 1 - F(L_B) = e^{-\lambda L_B} \cong 1 - \lambda L_{BX} \tag{1}$$

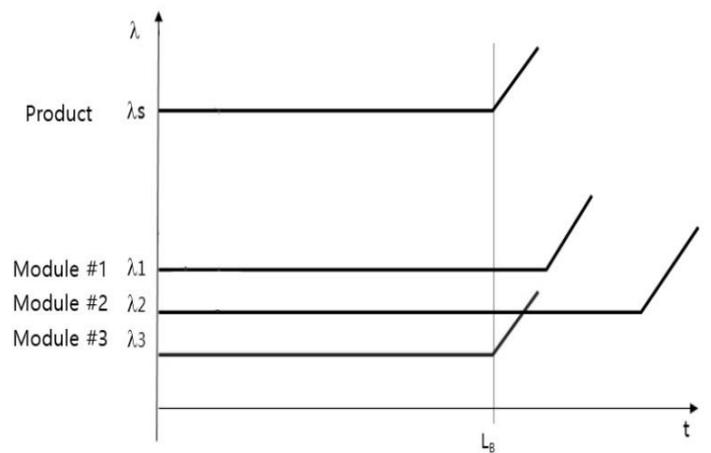
This equation is applicable below about 20 percent of cumulative failure [1]. Reliability design of the mechanical system can be achieved by getting the targeted product lifetime  $L_B$  and failure rate  $\lambda$ . As finding the faulty designs, engineers could modify the defective configurations of structures through an overall parametric accelerated life testing (ALT) plan and its implementation. Therefore, reliability concepts-parametric ALT plan, failure mechanism and design, acceleration factor, and sample size equation are discussed below.

**Setting overall parametric ALT plan of product**

As you can see in Figure 2a, a product can consist of several different modules. For example, automobiles consist of modules, such as the engine, transmission, drive, electrical, and body parts. Total components have approximately 25,000 pieces. Product reliability is targeted to B40~120 life 12 year. Suppose that there were no initial failures in a product, product lifetime could be determined by the module #3 in Figure 2. The cumulative failure rate of the product (automobile) over its lifetime would be the sum of the failure rate of each module as seen in Figure 2b. If one core module #3 has faulty designs, it will seriously damage the reliability of the whole product. Therefore, engineer will improve the design of module #3 by reliability testing.



(a). Breakdown of Automobile with multi-modules (Courtesy of Korea Testing Co., Inc.)



(b). Product lifetime  $L_B$  and failure rate  $\lambda_s$  with multi-modules

**Figure 2:** Breakdown of product and Product lifetime  $L_B$  and failure rate  $\lambda_s$

Table 1 shows the parametric ALT for several modules. In targeting the reliability of the mechanical product in parametric ALT, there are three cases for modules in mechanical product: 1) modified module, 2) new module, and 3) similar module to the prior design on the basis of market data. For module A, the expected failure rate was 0.34 %/yr and its expected lifetime was 5.3 years because there was no field data on the reliability of the new design. The reliability of the new design was targeted to be over B 13.2 life 12 years. To meet the expected product lifetime, the parametric ALT might help identify the faulty designs that could affect the product reliability.

No	Modules	Market Data		Design	Conversion	Expected		Targeted	
		Yearly Failure Rate, %/yr	$B_x$ Life, yr ( $x=1.8$ )			Yearly Failure Rate, %/yr	$B_x$ Life, yr ( $x=1.8$ )	Yearly Failure Rate, %/yr	$B_x$ Life, yr
1	Module A	0.34	5.3	New	x5	1.70	1.1	0.15	12( $B_x=1.8$ )
2	Module B	0.35	5.1	Given	x1	0.35	5.1	0.15	12( $B_x=1.8$ )
3	Module C	0.25	4.8	Modified Motor	x2	0.50	2.4	0.10	12( $B_x=1.2$ )
4	Module D	0.20	6.0	Modified	x2	0.40	3.0	0.10	12( $B_x=1.2$ )
5	Module E	0.15	8.0	Given	x1	0.15	8.0	0.1	12( $B_x=1.2$ )
6	Others	0.50	12.0	Given	x1	0.50	12.0	0.5	12( $B_x=6.0$ )
Total	R-Set	1.79	7.4	-	-	3.60	3.7	1.10	12( $B_x=13.2$ )

Table 1: Overall parametric ALT plan of product.

**Failure Mechanics, Design and Reliability Testing**

As mentioned, most products such as appliance, car, and aircraft are composed of multi-module structures. They may compose of multiple electrical or mechanical components. If these modules are put together, products have their own product function. For example, to store the food fresh, the refrigerator is designed to provide cold air from the evaporator to the freezer (or refrigerator compartment). Modules need to be robustly designed to withstand a variety of loads. In determining lifetime, the module robust design determines the control factor (or design parameters) to endure the noise factor (or stress) and properly work the system, which has the reliability target - part failure rate  $\lambda$  and lifetime  $L_B$  (Figure 3).

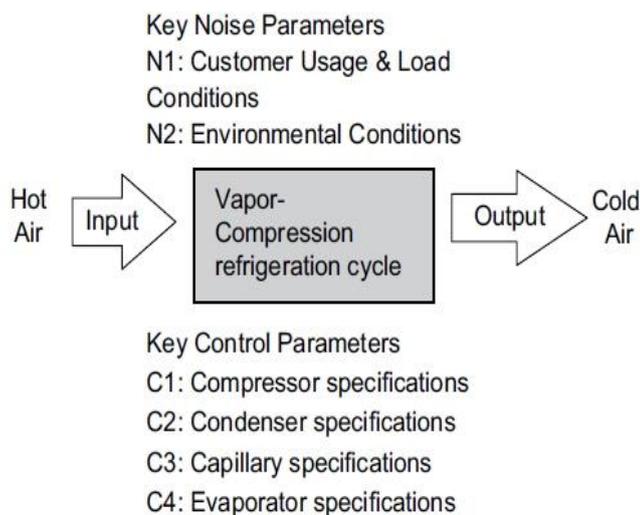


Figure 3: Typical robust design schematic (example: refrigerator).

If product structure is ideally designed and has well-dispersed stresses, there might be no problems in its lifetime. Though the mechanical design is designed

optimally by Finite Element Analysis (FEA), it may have design flaws that will show up in the field.

If there is a faulty design in the module structure where the repetitive loads are applied, the structure can fracture in its lifetime. The product engineer would want to find the faulty designs in the structure and correct them. Therefore, reliability testing should be validated before products launch.

Repeated loads or overloading due to daily consumer usage may cause structural failure in the product and reduce its lifetime. Many engineers think such possibilities can be assessed by: 1) mathematical modeling using Newtonian methods, 2) assessing the time response of the system for dynamic loads, 3) utilizing the rain-flow counting method [2], and 4) estimating system damage using the Palmgren-Miner’s rule [3]. However, because there are many assumptions, this analytic methodology may be exact, but complex to reproduce the product failures due to the design flaws (Figure 4).

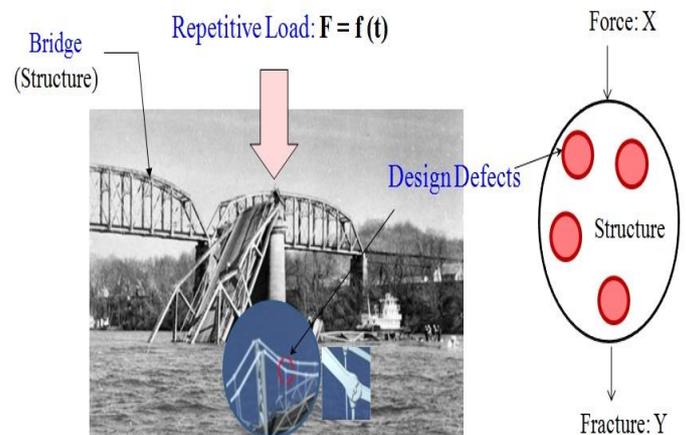


Figure 4: Failure mechanics created by a load on a component made from a specific material.

The reliability targeting is known to be conventionally achieved through the Taguchi methods (SDE) and the statistical design of experiment [4]. Taguchi methods use the loss function which quantifies the amount of loss based on deviation from the target performance. Taguchi's methods are known to produce robust designs. It puts a design factor in an optimal location where random "noise" factors are less likely to hurt the design and it helps determine the best control factors (or design parameters). However, for an uncomplicated mechanical structure, such as a beam, Taguchi methods should take into account a considerable number of design parameters. In the design process it is not possible to consider the whole range of the physical, chemical and the mathematical conditions that could affect the design.

In this study, we suggest a parametric Accelerated Life Testing (ALT) method that can improve the reliability of a mechanical product subjected to repetitive stresses. It will be another experimental methodology as alternatives. To determine the proper design parameters, parametric ALT used new sample size equations with an Accelerated Factor (AF), which will be used to confirm the final design.

## Materials & Methods

### Parametric Accelerated Life Testing in Mechanical System

Parametric accelerated life testing finds the faulty designs of product under the accelerated conditions. It uses the sample size equation with Acceleration Factor that engineers could find the optimal designs. It also can help them better estimate expected lifetime  $L_B$ , failure rate of module  $\lambda$ , and determine the overall product reliability. It therefore is important to derive the sample size equation with the whole parameters - lifetime  $L_B$ , acceleration factor  $AF$ , the actual testing time  $h_a$ , and the allowed number of failures  $r$ .

Under the expected physical and chemical conditions that the product is expected to experience in field, it is essential to derive the acceleration factor from a life-stress mode and determine the dominant failure mechanism for the product. A grasp of physical of failure (PoF) also is required to understand the failure mechanism. For example, fatigue or fracture due to repetitive stresses is the common mechanism for failure in mechanical system.

Reliability engineers must determine how the stresses (or loads) act on the product structure, which help categorize the potential failure mechanisms under the range in environmental and operational conditions. Engineers need to develop a testing plan with appropriate accelerated load conditions to determine the dominant failure mechanisms affecting product lifetime. At the same time, they also must include other failure mechanisms, such as overstress and wear out.

### Acceleration Factor (AF)

For solid-state diffusion of impurities in silicon, the junction equation  $J$  might be expressed as:

$$J = A \sinh(a\xi) \exp\left(-\frac{Q}{kT}\right) \quad (2)$$

On the other hands, reaction process that is dependent to speed might be expressed as:

$$K = a \frac{kT}{h} e^{-\frac{\Delta E}{kT}} \sinh\left(\frac{aS}{kT}\right) \quad (3)$$

So, the reaction rate  $K$  from equation (2) and (3) can be summarized as:

$$K = B \sinh(aS) \exp\left(-\frac{E_a}{kT}\right) \quad (4)$$

If the reaction rate in equation (4) takes an inverse number, the generalized stress model can be obtained as,

$$TF = A [\sinh(aS)]^{-1} \exp\left(\frac{E_a}{kT}\right) \quad (5)$$

where  $A$  and  $a$  are coefficients.

Because this life-stress model equation was derived from a model of micro-depletion (void) in the failure domain, it should be relevant to general failure prediction regardless of whether it is a mechanical or electronic system. Thus, the fatigue in a mechanical system, coil degradation in a motor, bond-pad corrosion in an IC, etc., can be captured by Equation (5).

The range of the hyperbolic sine stress term  $[\sinh(aS)]^{-1}$  in equation (5) is increasing the stress as following: 1) initially  $(S)^{-1}$  in low effect, 2)  $(S)^{-n}$  in medium effect, and 3)  $(e^{aS})^{-n}$  in high effect initially linearly increasing. Accelerated testing usually happens in the medium stress range. The hyperbolic sine stress term in the level of medium stress can be properly substituted with the power term  $(S)^{-n}$ . Time to failure can then be described as

$$TF = A(S)^{-n} \exp\left(\frac{E_a}{kT}\right) \quad (6)$$

The internal (or external) stress in a product is difficult to quantify and use in accelerated testing. It is necessary to modify equation 5 into a more applicable form. The power (or energy flow) in a physical system can generally be expressed as efforts and flows (Table 2). Thus, stresses in mechanical or electrical systems may come from the efforts (or loads) like force, torque, pressure, or voltage [5].

Modules	Effort, $e(t)$	Flow, $f(t)$
Mechanical translation	Force, $F(t)$	Velocity, $V(t)$
Mechanical rotation	Torque, $\tau(t)$	Angular velocity, $\omega(t)$
Compressor, Pump	Pressure difference, $\Delta P(t)$	Volume flow rate, $Q(t)$
Electric	Voltage, $V(t)$	Current, $i(t)$

**Table 2:** Energy flow in the multi-port physical system

For a mechanical system, when replacing stress with effort, the time-to-failure can be modified as

$$TF = A(S)^{-n} \exp\left(\frac{E_a}{kT}\right) = A(e)^{-\lambda} \exp\left(\frac{E_a}{kT}\right) \quad (7)$$

Because the material strength degrades slowly in many mechanical stress/strength interfaces, it may require long times to test a module until failure occurs. On the other hand, the product failures due to overstressing can be found with parametric ALT in the early stage. The main hurdles to finding wear induced failures and overstressed failures are the testing time and cost. To solve these issues, the reliability engineer often prefers testing under severe conditions.

The more the accelerated conditions, the shorter the testing time will be. This concept is critical to performing accelerated life tests, but the range of the accelerated life tests will be determined by whether the conditions in the accelerated tests are the same to that in normally found in the field.

In accelerated life tests, when a module has been tested for a number of hours under the accelerated stressed condition, one wants to know the equivalent operation time at the normal stress condition. The equivalent operation time is obtained from the multiplication of the acceleration factor and normal (or actual) operation time.

From the time-to-failure in equation 6, the acceleration factor can be defined as the ratio between the proper accelerated stress levels and typical operating conditions. The acceleration factor ( $AF$ ) can be modified to include the effort concepts:

$$AF = \left(\frac{S_1}{S_0}\right)^n \left[\frac{E_a}{k} \left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] = \left(\frac{e_1}{e_0}\right)^\lambda \left[\frac{E_a}{k} \left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right] \quad (8)$$

The first term is the outside effort (or load) and the second is the internal energy in equation (8). Under severe conditions, the outside higher load drops the energy barrier and the accelerated (or high) temperature activates the material elements. In the end, the material degrades and fails. The equation has two parameters which are temperature and effort. Using a three-level test under accelerated conditions, these parameters can be obtained. And the quantified value, activation energy (eV), is called the reaction rate due to temperature rises. Under severe conditions, the duty effect with repetitive stress (or load) involves the on/off cycles, which shortens module lifetime [6].

**Sample size equation**

To carry out parametric ALTs, the sample size equation with the acceleration factors in equation (8) might be expressed as [7]:

$$n \geq (r + 1) \cdot \frac{1}{x} \cdot \left(\frac{L_{BX}^*}{AF \cdot h_a}\right)^\beta + r \quad (9)$$

If the reliability of the mechanical system was targeted, number of required test cycles (or mission cycles) can be obtained for given sample size. Through parametric ALTs the faulty designs of mechanical system can be identified to achieve the reliability target.

**Case Study - Freezer Drawer in a French Door Refrigerator Subjected to Repetitive Food Loading**



**Figure 5:** French Door Refrigerator and freezer drawer assembly. (a) French Door Refrigerator, b) Mechanical parts of the drawer: 1) handle, 2) drawer, 3) slide rail, and 4) pocket box.

Figure 5 shows a French door refrigerator with a new freezer drawer system. The consumers want to keep their food fresh for a period of use time and have convenient access to the food in the refrigerator. Engineers have designed the French door refrigerator to handle the required food storage loads found under expected consumer usage conditions. Storing food in the freezer drawer therefore has two repetitive handling steps: 1) opening the drawer to store food in the drawer, and 2) taking food out of the drawer.

We found that the freezer drawer was being returned with fractures from consumers in the field and requiring replacement of the refrigerator. When subjected to repetitive stresses during customer usage, the root causes of the failed freezer drawer came from a problem in the design of the drawer. Analysis of the returned products also showed that the freezer drawer had critical design flaws in its structure. Thus, the freezer drawer had to be redesigned to withstand repetitive loads under customer usage conditions and improve its reliability.

The consumer stores the food in the freezer drawer in order to keep the stored food fresh. The concentrated stress due to the food mechanical load is applied on the freezer drawer and its sliding rails. When designing the freezer drawer, it was critical to design it so as to ensure that the drawer could withstand these repetitive mechanical loads from the opening/closing of the drawer.

For the freezer drawer system in French door refrigerator, the working conditions of customer ranged from 0 to 43°C with a relative humidity ranging from 0% to 95%, and 0.2 to 0.24g's of acceleration. In the United States, the operating cycles of the freezer drawer depend on consumer usage. Data showed that consumers opened and closed the drawer system of a French door refrigerator between five and nine times a day. The closing and opening of the Freezer drawer happens approximately 5 to 9 times per day. With a life cycle design point for 10 years, Freezer drawer system incurred about 36,500 usage cycles (Figure 6).

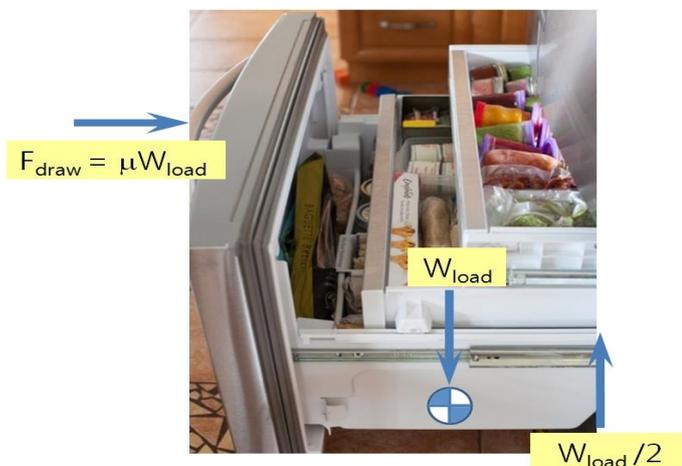


Figure 6: Functional design concept of the freezer drawer system.

The force balance at the free-body diagram of the freezer drawer system can be expressed as:

$$F_{draw} = \mu W_{load} \tag{10}$$

Because the stress of the freezer drawer system depends on the applied force in proportion to foods weight, the time-to-failure from equation (6) can be represented as:

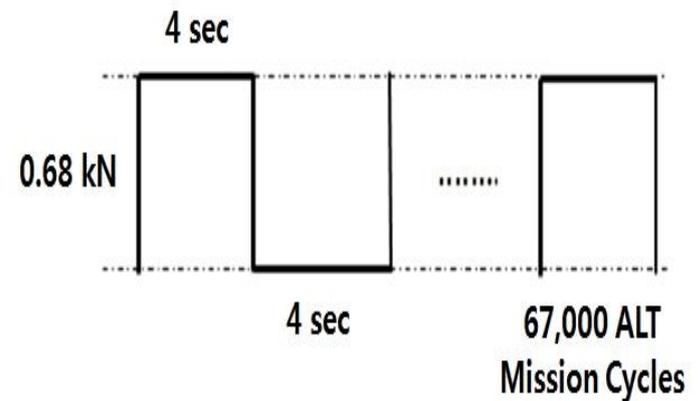
$$TF = A(S)^{-n} = A(F_{draw})^{-\lambda} = A(\mu W_{load})^{-\lambda} \tag{11}$$

The acceleration factor (AF) can be expressed as:

$$AF = \left(\frac{S_1}{S_0}\right)^n = \left(\frac{F_1}{F_0}\right)^\lambda = \left(\frac{\mu W_1}{\mu W_0}\right)^\lambda = \left(\frac{W_1}{W_0}\right)^\lambda \tag{12}$$



(a) ALT equipment controller.



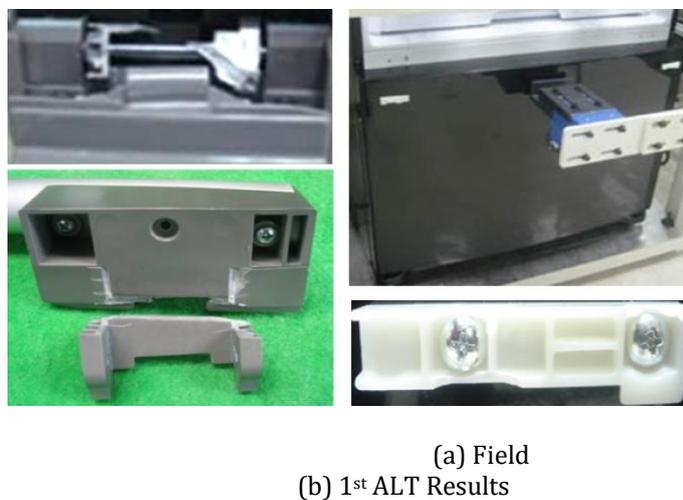
(b) Duty cycles of repetitive food weight force on the drawer

Figure 7: ALT equipment and duty cycles.

For the worst case of the food weight, the required force on the handle of the Freezer drawer was 0.34kN. The applied food weight force for the accelerated life testing was

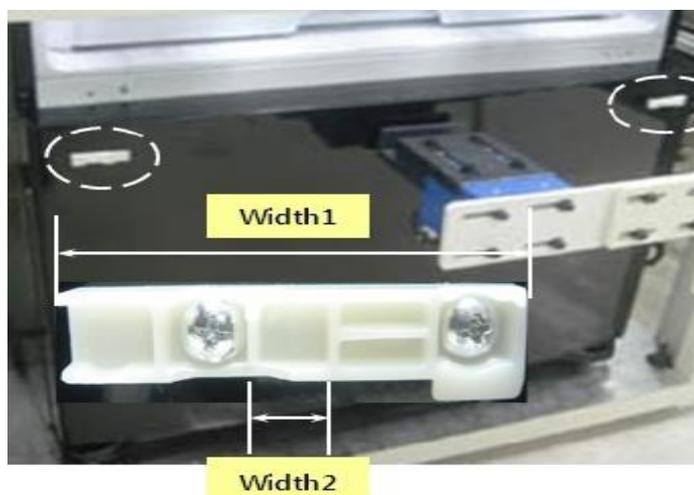
0.68kN. With a quotient,  $\lambda$ , of 2, the total  $AF$  was approximately 4.0 using equation (12). Suppose that the shape parameter was 2.0 and the allowed failed numbers  $r$  was 0, then the test time and the number of samples calculated from equation (9) would be 67,000 and 3 units for the first parametric ALT. To meet the reliability target B1 life 10 years, there needs to be no fractured sample in 67,000 cycles that might be the reliability test specifications (Figure 7).

In 1<sup>st</sup> ALT the handle of the drawer fractured at 7,000 cycles and 8,000 cycles. Figure 8 shows the failed product from the field and the 1<sup>st</sup> accelerated life testing, respectively. As seen in field and 1<sup>st</sup> ALT, they are very similar. If the freezer drawer is subjected to repetitive opening and closing with the food load, we could conclude that its design flaws at the right place could result in a fracture. By parametric ALT, we reproduced the faulty handle structure of Freezer drawer.



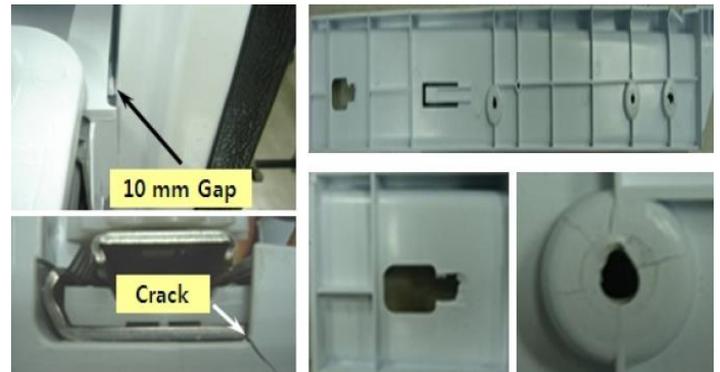
**Figure 8:** Failure of Freezer drawer handles in the field and 1<sup>st</sup> ALT result.

To withstand the drawer fracture due to the repetitive food stresses, the handle was redesigned as follows: (1) increasing the width of the reinforced handle, C1, from 90mm to 122mm; (2) increasing the handle hooker size, C2, from 8mm to 19mm (Figure 9).

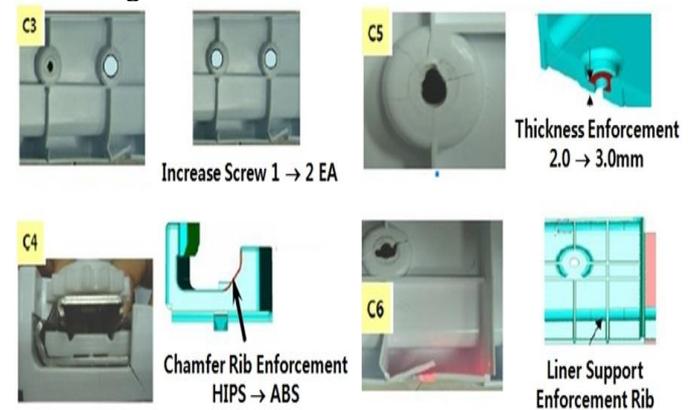


**Figure 9:** Redesigned Freezer drawer handle

In 2<sup>nd</sup> ALTs the slide rails of the drawer cracked at 15,000 cycles and 16,000 cycles (Figure 10). When the food weight force for the accelerated life testing is applied repetitively, the weakest part -slide rail reveals. The root cause came from the corner shape of the slide rails. Consequently, it started to crack and fractures in the end. As corrective action, the slide rail is 1) increasing the rail fastening screw number, C3, from 1 to 2; 2) adding an inner chamber and plastic material, C4, from HIPS to ABS; 3) thickening the boss, C5, from 2.0mm to 3.0mm; 4) adding a new support rib, C6. By parametric ALT, we can modify the faulty slide rail of the Freezer drawer (Figure 11).



**Figure 10:** Failed slide rails in second ALT.



**Figure 11:** Redesigned Slide Rail.

As the problematic designs were improved over the course of the two ALTs, there were no design problems with the freezer drawer until 65,000 cycles in the third ALT. We therefore concluded that the modified design parameters found from the 1<sup>st</sup> and 2<sup>nd</sup> ALT were effective in improving the reliability design of the Freezer drawer.

## Conclusions

To accomplish the reliability design of mechanical products, the basic concepts of parametric ALT were discussed: -1) Setting overall parametric ALT plan of product, 2) Failure mechanics, design and reliability testing, 3) Parametric accelerated life testing with action plans, and 4) Checking if final design of product is achieving the reliability target. The failure modes and mechanisms of the mechanical system in the field and parametric ALT may come from the design flaws not considered in the design

phase. As case study, we have studied the reliability of a newly designed Freezer drawer.

In the first ALT, the handles were being fractured because of design flaws due to repetitive opening/closing with the food loads. As the total handle width increases, the handle design was corrected. During the second ALT, the slide rails also fractured because they did not have enough strength to withstand repetitive opening and closing of the drawer with the food loads. Additional reinforced ribs, reinforced boss, and an inner chamber in slide rails were provided which improved the design of the slide rails. Other case studies on the design flaws were suggested in references 9 through 19.

Based on the failed products from the marketplace and the sample results of the parametric ALTs, the product with the modified designs might require reliability testing to check whether it meet the reliability target. With the study of the faulty designs for the mechanical system, the parametric ALTs can be effective in proving a more reliable product with significantly longer life. Under consumer usage conditions, this reliability design methodologies also will provide the reliability test specifications of a mechanical structure subjected to repetitive stresses.

These reliability methodologies are applicable to other mechanical systems-refrigerators, automobiles, construction equipment, washing machines, vacuum

cleaners, and the construction structure of civil engineering. For improving the reliability design of these systems, the faulty designs need to be identified to meet the product reliability.

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